M.TECH. (III SEMESTER) EXAMINATION
(MECHANICAL ENGINEERING)
FINITE ELEMENT METHODS
(ME-675)

Maximum Marks: 60
Credits: 04
Duration: Three Hours

Answer five (05) questions.
Question one (1) is compulsory.
Assume suitable data if missing.
Notations used have their usual meaning.

Q.No. Question M.M.

1(a) State and explain C⁰ and C¹ element continuity. [03]

1(b) Define the weak form of weighted residual. [03]

1(c) Explain Isoparametric representation for displacement field. [03]

1(d) Prove that area of the triangle \( A = \frac{1}{2} |\text{det}J| \), where \( J \) is Jacobian. [03]

2(a) Determine a two parameter Galerkin’s approximation solution of the differential equation.
\[
\frac{d^2 u}{dx^2} = f_0, \quad 0 < x < L \\
\frac{du}{dx} = 0 \quad \text{at} \quad x = 0 \text{ and } L.
\] [06]

2(b) Derive the stiffness matrix (\( K \)) for a typical quadratic bar element assuming area of cross section \( A_e \), Length \( L_e \) and Young’s Modulus \( E_e \). [06]

3 An axial load \( P = 300 \times 10^3 \) N is applied at 20° C to the rod shown in Fig.1. The temperature is then raised to 60° C. Assemble the stiffness and force matrices. Determine the nodal displacements, element stresses and support reactions. Solve the problem using elimination method for handling the boundary conditions. [12]

4 A uniform cantilever beam subjected to 1000 N load at free end, having length 0.5m and cross-sectional area 0.1 m x 0.06 m. The young’s modulus for beam is 69Gpa and poisons ratio (\( \nu \)) = 0.33. Considering one element and 2-dof at each node, obtain slope and deflection at free end. [12]
5(a) For the 2-D plane truss, derive the elemental stiffness matrix $K$ in term of usual notations.

5(b) For the pin-jointed configuration shown in Fig.2, determine the stiffness values $K_{11}$, $K_{12}$ and $K_{22}$ of the global stiffness matrix.

6(a) Fig.3 shows a four noded quadrilateral element. The (x,y) coordinates of each node are given in the figure. The element displacement vector $q = [0, 0, 0.20, 0, 0.15, 0.10, 0, 0.05]^T$. Find the following.
   
   i. The x-y coordinates of a point P, whose location in the master element is given by $\xi = 0.5$ and $\eta = 0.5$

   ii. The u, v displacements of the point P.

6(b) Using a $2 \times 2$ rule, evaluate the integral $\iint_A (x^2 + xy^2) \, dx \, dy$ by Gaussian quadrature, where $A$ denotes the region shown in Fig.3.

7(a) A composite wall consists of three materials as shown in Fig.4. The outer temperature is $T_o = 20^\circ C$. Convection heat transfer takes place on the inner surface of the wall with $T_\infty = 800^\circ C$ and $h = 25\, W/m^2\cdot ^\circ C$. Considering three linear elements, determine the temperature distribution in the wall.

7(b) Consider the shaft with a rectangular x - section shown in Fig.5. Using FEM, model one fourth of the shaft and determine the angle of twist per unit length in terms of $M$ and $G$.

8 Evaluate the lowest eigenvalue and the corresponding eigenmode for the beam shown in Fig.6. Consider two linear beam elements and 2- dof at each node.