1. (a) Consider the density function:

\[
 f(x) = \begin{cases} 
 k\sqrt{x}, & 0 < x < 1 \\
 0, & \text{elsewhere}
\end{cases}
\]

(i) Evaluate \( k \) and (ii) find \( P(0.3 < x < 0.6) \)

(b) If the probability that a fluorescent light has a useful life of at least 800 hours is 0.9, find the probabilities that among 20 such lights at least 15 will have a useful life of at least 800 hours.

(c) A secretary makes 2 errors per page, on average. What is probability that on the first page he or she will make? (i) 4 or more errors (ii) no errors.

2. (a) A process for making certain bearings is under control if the diameters of the bearings have a mean of 0.500 cm with level of significance 0.05. What can we say about this process if a sample of 10 of these bearings has a mean diameter of 0.5060 cm and a standard deviation of 0.0040 cm?

(b) A random sample of 10 observations is taken from a normal population having variance 42.5. Find the approximate probability of obtaining a sample standard deviation between 3.14 and 8.94.

(c) If two independent samples of sizes 26 & 8 are taken from a normal population, what is the probability that the variance of the second sample will be at least 2.4 times the variance of the first sample?

3. (a) A manufacturer of sports equipment has a new synthetic fishing line that he claims has a mean breaking strength of 8 kg with standard deviation of 0.5 kg. Test the hypothesis that \( \mu = 8 \) kg against the alternative hypothesis that \( \mu \neq 8 \) kg if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kg. Use a 0.01 level of significance. The standard deviation of the sample was 0.8 kg.
(b) Five samples of a ferrous -type substance are to be used to determine if there is a difference between a laboratory chemical analysis and a X-ray analysis of the iron content. Each sample was split into two samples and the two types of analysis were applied. Following are the coded data showing the iron content analysis. Find if there is no difference in both analyses.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-ray</td>
<td>2.0</td>
<td>2.0</td>
<td>2.3</td>
<td>2.1</td>
<td>2.4</td>
</tr>
<tr>
<td>Chemical</td>
<td>2.2</td>
<td>1.9</td>
<td>2.5</td>
<td>2.3</td>
<td>2.4</td>
</tr>
</tbody>
</table>

4. Four kinds of fertilizers $f_1, f_2, f_3, f_4$ are used to study the yield of beans. The soil is divided into 2 blocks containing 4 homogeneous plots. The yields in kg per plot and the corresponding treatments are as follows:

<table>
<thead>
<tr>
<th>Block 1</th>
<th>$f_1 = 42.7$</th>
<th>$f_2 = 48.3$</th>
<th>$f_3 = 32.8$</th>
<th>$f_4 = 39.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block 2</td>
<td>$f_1 = 50.9$</td>
<td>$f_2 = 50.0$</td>
<td>$f_3 = 38.0$</td>
<td>$f_4 = 40.2$</td>
</tr>
</tbody>
</table>

Conduct an analysis of variance at 0.05 level of significance using RCB design.

5. Explain the procedure of Latin Square design with the suitable example. Also construct the ANOVA table for $\Gamma \times \Gamma$ Latin Square.

6. An experiment was conducted to increase the adhesiveness of rubber products. Four products were made with new additive and another four without the new additive. The observed adhesiveness is recorded below:

<table>
<thead>
<tr>
<th>Temperature ($^\circ$C)</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without additive</td>
<td>2.3</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>2.9</td>
<td>3.7</td>
</tr>
<tr>
<td>With additive</td>
<td>4.3</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td>3.9</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Perform an analysis of variance to test for significant main and interaction effects.

7. The amount of chemical compound $\gamma$ which dissolved in 100 gm of water at various temperatures $x$ were recorded as follows:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>8</td>
<td>12</td>
<td>25</td>
<td>31</td>
</tr>
</tbody>
</table>

(i) Find the equation of regression line
(ii) Evaluate mean squared error $s^2$.
(iii) Construct 99% confidence intervals for $\alpha$ and $\beta$. 

10

15

15

15
1. (a) Name any two combinations of refrigerant-absorbent and their applications. Which combination is used in the air-conditioning application? Give reasons in support of your answer.

(b) Why are analyzer and rectifier employed in the ammonia – water absorption system but not in a lithium bromide – water system?

(c) What is the role of a heat exchanger in a vapour absorption cycle? Draw a schematic diagram of an absorption cycle with heat exchanger and explain how it improves the operational efficiency.

2. (a) A refrigerator using refrigerant R-134 a as a working fluid operates on an ideal vapour compression cycle between 0.12 and 0.7 MPa. The mass flow rate of the refrigerant is 0.06 kg/s. Show the cycle on a T-S diagram and determine the following:

(i) The rate of heat removal of the refrigerated space and the power input to the compressor.

(ii) The rate of heat rejection to the environment and

(iii) The coefficient of performance

(b) A single compressor using R- 12 as refrigerant has three evaporators of capacity 10 TR, 20 TR and 30 TR. All the evaporators operate at – 10\(^0\)C and the vapours leaving the evaporators are dry and saturated. The condenser temperature is 40\(^0\)C. The liquid refrigerant leaving the condenser is sub cooled to 30\(^0\)C. Assuming isentropic compression, find (i) the mass of refrigerant flowing through each evaporator (ii) the power required to drive the compressor and (iii) the COP of the system.

3. (a) With the aid of a line and block diagram, describe the operation of Linde- Hampson system of liquefying air. Draw the cycle on a T-S diagram.

(b) Explain the process of adiabatic demagnetization.
4. (a) Briefly describe the biomedical applications of cryogenics. Describe the process of liquefaction of helium. 
(b) Describe the Claude process of air liquefaction. Show the process on T-S diagram. 

5. (a) Draw a block diagram of thermoelectric refrigerator and indicate how different components of this cycle correspond to those of the vapour compression cycle. 
(b) Describe the principle of operation of a steam jet water vapour refrigeration system. What are its advantages and limitations?

6. (a) Describe the system of nomenclature of refrigerants. 
(b) A two stage ammonia system designed for a fish freezing plant uses flash gas removal and inter-cooling operation. The condensing temperature is $35^\circ$C. The saturation temperature of the high temperature evaporator is $0^\circ$C and its capacity is 90 kW. The saturation temperature of the low temperature evaporator is $-40^\circ$C and its capacity is 170 kW of refrigeration. Draw the cycle on a p–h diagram and determine the following:
   (i) the rate of refrigerant compressed by the high-stage compressor 
   (ii) the power required by the compressors, and 
   (iii) the coefficient of performance

7. (a) What are the desirable properties of a good refrigerant? 
(b) Write short notes on the following:
   (i) Azeotropes 
   (ii) Secondary refrigerant 
   (iii) Air refrigeration 
   (iv) Retrofitting 
   (v) Montréal and Kyoto Protocol
2011-2012
M.TECH. (III SEMESTER) EXAMINATION
(MECHANICAL ENGINEERING)
COMPUTATIONAL METHODS IN THERMAL ENGG
(ME - 668)

Maximum Marks: 75
Duration: Three Hours

Note:
1. Answer all questions.
2. Assume suitably any missing data.
3. Symbols have their usual meanings.

1. (a) Derive the central difference approximation for \( \frac{d^2f}{dx^2} \) of order 2. (05)
(b) Derive the central difference approximation for \( \frac{df}{dx} \) using backward difference of order 2. (05)
(c) Given the following data, compute \( \frac{df}{dx} \) at \( x = 9 \) and \( \frac{d^2f}{dx^2} \) at \( x = 6 \). Use 2\(^{nd}\) order accurate finite difference schemes. (05)

<table>
<thead>
<tr>
<th>x</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>150</td>
<td>252</td>
<td>392</td>
<td>576</td>
<td>810</td>
<td>1044</td>
</tr>
</tbody>
</table>

2. (a) Perform Von Neumann stability analysis to determine stability of FTFS explicit and implicit schemes for the linear advection equation. Also check for consistency and convergence of these schemes. (10)
(b) Suppose you are given samples \((x_0, f(x_0)), (x_0 + h, f(x_0 + h)), (x_0 + 2h, f(x_0 + 2h))\) and \((x_0 + 3h, f(x_0 + 3h))\). Approximate \( \frac{df}{dx} \) at \( x_0 + h \) and \( \frac{d^2f}{dx^2} \) at \( x_0 + 2h \). OR

2'. (a) Perform Von Neumann stability analysis to determine stability of leap-frog scheme for the linear advection equation. Also check for consistency and convergence of these schemes. (08)
(b) Given the PDE, \( \frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} \), discuss the difference between fully explicit and fully implicit formulations. Also list the advantages and limitations of each of these formulations. (07)

3. Explain what do you understand by artificial viscosity. Consider linear advection equation and discretize it using FTFS and FTBS schemes. Find artificial viscosity for both schemes and discuss the relationship between artificial viscosity and stability of both schemes. Check the convergence of both schemes. (15)

Contd.....2
3. (a) Given the PDE, \( \frac{\partial^2 u}{\partial t^2} = a \frac{\partial u}{\partial x} + b \frac{\partial^2 u}{\partial x^2} \), discretize this equation using fully implicit scheme with central difference for time, forward for first derivative and central difference for the second derivative. State the order of truncation error. Assume \( a \) and \( b \) as constants.

(b) Apply a Taylor series expansion to a mixed backward formula for the first derivative: \( \left( u_x \right)_i = \left( a u_{i-2} + b u_{i-1} + c u_i + d u_{i+1} \right) / \Delta X \). Derive the family of 2\(^\text{nd}\) order accurate formulas and the corresponding truncation error as function of \( d \).

4. A hydrodynamically fully-developed, thermally developing laminar flow inside a circular pipe is governed by non-dimensional heat equation

\[
\text{Pe} \, u(r) \frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}.
\]

The flow velocity is expressed as

\[
u(r) = \frac{3}{2} \left( 1 - r^2 \right).
\]

(a) Discretize the governing equation using first order upwind scheme for the convective term and 2\(^\text{nd}\) order accurate central difference scheme for diffusion terms.

(b) Write down the discretized form of all the boundary conditions especially the axisymmetric condition at centre-line of the pipe.

(c) Write a pseudo-code showing the logical arrangement of loops, updating of boundary conditions, convergence criteria etc.

5. We wish to solve Navier-Stokes and continuity equations in a shear-driven 2D lid cavity problem using SMAC scheme on a collocated grid. The boundary conditions are no-slip at the walls except the top wall has \( u = 1 \) and \( v = 0 \).

(a) Write down the discretized form of governing equations using QUICK scheme for the convective terms.

(b) Write down the discretized form of Passon equation using SOR scheme and enumerate the value of relaxation factor \( \omega \) to be used.

(c) Write down a pseudo-code showing logical arrangement of space loops, updating of boundary conditions, time loops, convergence criteria, etc.

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2011-2012
M.TECH. (III SEMESTER) EXAMINATION
(MECHANICAL ENGINEERING)
FINITE ELEMENT METHODS
(ME - 675)

Maximum Marks: 75

Note:
1. Answer any five questions.
2. Missing data if any, may be assumed suitably.
3. Symbols used have their usual meanings.

1. Consider the bar shown in Fig. 1. An axial load $P = 200 \times 10^3$ N is applied as shown. Considering two finite elements and d.o.f. at each mode 1, determine the nodal displacements, the stresses in each material and the reaction forces.

2. (a) Consider the functional $I$ for minimization given by

$$I = \int K \left( \frac{dy}{dx} \right)^2 dx + \frac{1}{2} h(a_0 - 800)^2$$

with $y = 20$ at $x = 60$. Given $K = 20$, $h = 25$ and $L = 60$, determine $a_0$, $a_1$ and $a_2$ using the polynomial approximation $y(x) = a_0 + a_1 x + a_2 x^2$ in the Rayleigh-Ritz method.

(b) Use Galerkin's method to find the displacement at the midpoint of the rod shown in fig. 2.

3. (a) Derive element stiffness matrix $\mathbf{K}^e$ and element load Vector $\mathbf{F}^e$ for a quadratic element. Consider single d.o.f. at each node. Use usual notation.

(b) Derive thermal load Vector $\mathbf{\theta}^e$ as

$$\mathbf{\theta}^e = \frac{E_e A_e l_e \alpha \Delta T}{(x_2 - x_1)} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

for thermal stress produced due to rise of temperature $\Delta T$. $E_e$, $A_e$, $l_e$ are Young's modulus, area and length of element. $\alpha$ is thermal expansion co-efficient, $(x_2 - x_1)$ is total length.

4. (a) Derive two-point Gaussian quadrature formula. Using above formula calculate weights and their locations.

(b) For the truss shown in fig. 3, a horizontal load of $P = 4000$ N is applied in the x-direction at mode 2. Write down the element stiffness matrix $\mathbf{K}^e$ for each element. Assemble the $\mathbf{K}$ matrix. Take $E = 210$ GPa, $A = 5$ cm$^2$.

5. A Cantilever beam of length 1m, and has a Cross-Sectional area ($0.1 \times 0.06$) m$^2$. The beam undergoes static deflection by a downward load $P = 2000$ N applied at free end. The beam is made of a material with $E = 69$ GPa and $\nu = 0.3$. Considering one finite element and d.o.f. two at each node. Calculate maximum slope and deflection for the beam.

Contd.......2
6. (a) Evaluate the shape functions \( N_1, N_2 \) and \( N_3 \) at the interior point \( P \) for the triangular element shown in Fig. 4. Also calculate Jacobian for the element.

(b) State and explain isoparametric representation for the triangular element, with the help of \((x,y)\) and \((\xi, \eta)\) Co-ordinates.

7. (a) Consider a rectangular element shown in Fig. 5. Assume Plane Stress condition \( E = 210 \) GPa, \( \nu = 0.3 \). Evaluate \( J, B \) and \( \sigma \) at \( \xi = 0 \), and \( \eta = 0 \). Take \( q = [0, 0, 0.002, 0.003, 0.006, 0.0032, 0, 0]^T \)

(b) Considering axisymmetric solid subjected to axisymmetric loading, write general equation of Potential energy. Also obtain Galerkin formulation for the same.

8. A composite wall consists of three materials as shown in Fig. 6. The outer temperature is \( T_0 = 20^\circ\text{C} \). Convection heat transfer takes place on the inner surface of the wall with \( T_\infty = 800^\circ\text{C} \) and \( h = 25 \text{ W/m}^2\cdot^\circ\text{C} \). Considering three linear elements. Determine the temperature distribution in the wall.

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Figures enclosed.