SHEET NO: 1

2015-2016

B.TECH. AUTUMN (III SEMESTER) EXAMINATION
(ELECTRONICS ENGINEERING)
HIGHER MATHEMATICS - I
AM 251
Credits-04

Maximum Marks: 60
Duration: Three Hours

Note:-
1. Answer all the questions.
2. Standard Normal Table is attached (SHEET NO: 4)

Q1. (a) (i) Evaluate by using Laplace transform
\[ \int_0^\infty e^{-t} \left( \frac{\sin t}{t} \right) dt \]

(ii) Find the inverse Laplace transform of \( \operatorname{col}^{-1} \left( \frac{s+a}{b} \right) \).

OR (a') (i) Find the Laplace transform of \( f(t) = t^2, \ 0 < t < a \) given that \( f(t + a) = f(t) \).

(ii) Find the inverse Laplace transform of \( \frac{e^{-as}}{s(s-2)} \).

(b) Solve by Laplace transform method:
\[ y(t) = t + \int_0^t y(\lambda) \sin(t - \lambda) d\lambda. \]

(c) Solve the differential equation by Laplace transform method:
\[ \frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 2y = r(t) \]
where \( r(t) \) is defined as:
\[ \begin{cases} 10 \sin 2t, & \text{if } 0 < t < \pi \\ 0, & \text{if } t > \pi \end{cases} \]
with the conditions \( y(t) = 1, \frac{dy}{dt} = -5 \) at \( t = 0 \).

Q2. (a) Find the directional derivative of \( \vec{V}. (\vec{V} \cdot \phi) \) at the point \( (1, -2, 1) \) in the
direction of the normal to the surface \( xy^2z = 3x + z^2 \) where
\( \phi = 2x^3y^2z^4 \) and \( \vec{V} \) is usual vector differential operator.

Contd.....2.
OR (a') Find the most general differentiable function \( f(r) \) so that \( \nabla f(r) \) is solenoidal, where \( r \) has its usual meaning.

(b) Determine the constants \( a \) and \( b \) so that the vector field
\[
\vec{A} = (2xy + 3yz)\hat{i} + (x^2 + axz - 4x^2)\hat{j} + (3xy + 2xyz)\hat{k}
\]
is irrotational and hence find the scalar function \( \phi \) such that \( \vec{A} = \nabla \phi \) given that \( \phi(1, -1, 2) = 4 \).

(c) Verify curl curl \( \vec{A} = \nabla \times (\nabla \times \vec{A}) = -\nabla \times \vec{A} \)

for \( \vec{A} = xz^2\hat{t} - 2x^2yz\hat{j} + 2xyz\hat{k} \).

Q3. (a) Verify Gauss divergence theorem for \( \vec{F} = (2x^2 - 3y)\hat{i} - 2xy\hat{j} - 4x\hat{k} \)
taken over the surface \( S \) bounded by the co-ordinate planes and the plane \( 2x + 2y + z = 4 \).

(b) Use Stokes's Theorem to evaluate \( \iint_S \vec{V} \cdot d\vec{S} \) taken over the upper portion of the surface \( x^2 + y^2 - 2ax + az = 0 \) and the bounding curve lies in the plane \( z = 0 \) when \( \vec{A} = (y^2 + z^2 - x^2)\hat{j} + (x^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - z^2)\hat{k} \).

OR (b') Apply Green's Theorem in the plane to show that the area bounded by a simple curve \( C \) is given by \( \int_C (\partial y - \partial x) \) and hence evaluate the area enclosed by the ellipse \( 4x^2 + 9y^2 = 36 \).

Q4 (a) Suppose that we have two urns, 1 and 2, each with two drawers. Urn 1 has a gold coin in one drawer and a silver coin in the other drawer, while urn 2 has a gold coin in each drawer. One urn is chosen at random; then a drawer is chosen at random from the chosen urn. The coin found in this drawer turns out to be gold. What is the probability that the coin came from urn 1?
(b) Suppose that twice as many items are produced (per day) by machine 1 as by machine 2. However, about 4 percent of the items from the machine 1 tend to be defective while machine 2 produces only about 2 percent defectives. Suppose that the daily output of the two machines is combined. A random sample of 10 is taken from the combined output. What is the probability that this sample contains 2 defectives?

OR

(b') The diameter on an electric cable, say $X$, is assumed to be a continuous random variable with probability density function given by:

$$f(x) = kx(1-x), 0 \leq x \leq 1$$

(i) Determine the constant $k$.

(ii) Determine a number $b$ such that $P(X < b) = 2P(X > b)$

(iii) Compute $P\left(\frac{1}{3} \leq X < \frac{2}{3}\right)$.

(iv) Compute $E(X)$.

(c) Suppose that the life lengths of two electronic devices, say $D_1$ and $D_2$, have distributions $N(40, 36)$ and $N(45, 9)$, respectively. If the electronic device is to be used for a 45-hour period, which device is to be preferred? If it is to be used for a 48-hour period, which device is to be preferred?
Standard Normal Distribution Table

\[ \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} \, du = P(Z \leq z) \]

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<tr>
<th>( z )</th>
<th>( 0 )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
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<td>0,0</td>
<td>0.5000</td>
<td>0.5040</td>
<td>0.5080</td>
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<td>0.5160</td>
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<td>0.6915</td>
<td>0.6950</td>
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<td>0.7054</td>
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<td>0.8315</td>
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<td>0.8389</td>
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<td>0.8523</td>
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<td>0.9651</td>
<td>0.9671</td>
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<td>0.9835</td>
<td>0.9851</td>
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<td>0.9897</td>
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<td>0.9964</td>
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<td>0.9994</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
</tr>
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<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>1.0000</td>
</tr>
<tr>
<td>2,0</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
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<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
1. (a) If \( w = f(z) \) is a regular function of \( z \) such that \( f'(z) \neq 0 \), then show that

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \log |f'(z)| = 0.
\]

(b) If \( \phi(z_0) = \int_C \frac{2z^2 + 7z + 1}{z - z_0} \, dz \) where \( C \) is the circle \( |z| = 2 \) and \( z_0 \) is inside \( C \), then evaluate \( \phi'(1-i) \).

(c) If \( u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x} \) and \( f(z) = u + iv \) is an analytic function of \( z \), then find \( f(z) \) such that \( f(2/2) = 0 \).

2. (a) Expand \( f(z) = \frac{z}{(z^2 - 1)(z^2 + 4)} \) in the Laurent series in the annular region \( 1 < |z| < 2 \).

(b) Apply calculus of residues and evaluate \( \int_{C} \frac{x^2}{(x^2 + 1)(x^2 + 4)} \, dx \).

(c) State Residue Theorem. Find the residues of \( f(z) = \frac{z^3}{(z-1)^3(z-2)(z+3)} \) at its poles and hence evaluate \( \int_{C} f(z) \, dz \), where \( C \) is \( |z| = 4 \).

3. (a) Show that a positive real root of the equation \( x^3 - 1 - \sin x = 0 \) lies in the interval \( \left[ 1, \frac{\pi}{2} \right] \). Write this equation in the form \( x = \phi(x) \) and apply general iteration method \( x_{n+1} = \phi(x_n) \), \( n = 0, 1, 2, \ldots \) to obtain \( x_4 \). Take \( x_0 = 1.2 \). Also show that the function \( \phi(x) \) considered by you satisfies the convergence condition \( |\phi'(x)| < 1 \) for all \( x \) lying in the interval \( \left[ 1, \frac{\pi}{2} \right] \).

(b) Write the following system of equations:

\[
\begin{align*}
x + 2y + 5z &= 0 \\
5x + 2y + z &= 12 \\
x + 4y + 2z &= 15
\end{align*}
\]

Contd......2
in the matrix $AX = B$ such that matrix $A$ is diagonally dominant. The perform 4
iterations of Gauss-Seidel method to find the solution $X^{(0)} = \begin{pmatrix} x^{(0)} \\ y^{(0)} \\ z^{(0)} \end{pmatrix}$. Consider the
initial solution vector $X^{(0)} = \begin{pmatrix} x^{(0)} \\ y^{(0)} \\ z^{(0)} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

(b) From the values given in the table below:

<table>
<thead>
<tr>
<th>θ in degrees</th>
<th>31</th>
<th>32</th>
<th>34</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>tan θ</td>
<td>0.5008</td>
<td>0.6249</td>
<td>0.6494</td>
<td>0.6745</td>
</tr>
</tbody>
</table>

Approximate tan θ

(i) at $θ = 31.5^0$ by Lagrange's polynomial

(ii) at $θ = 34.5^0$ by Divided Difference polynomial

4. (a) The following table gives the values of a function $f(x)$ at equal intervals.

<table>
<thead>
<tr>
<th>x</th>
<th>0.0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = f(x)</td>
<td>0.3989</td>
<td>0.3521</td>
<td>0.2420</td>
<td>0.1295</td>
<td>0.0540</td>
</tr>
</tbody>
</table>

Find the volume of revolution when the curve $y = f(x)$ between $x = 0.0$ to $x = 2.0$
is resolved about $x$-axis.

(b) Solve the initial value problem

$$\frac{dy}{dx} = -x^2 + y^2, \quad y(0) = 0$$

by using fourth order Runge-Kutta method and obtain $y(0.4)$ considering $h = 0.2$.

OR

(b') Solve

$$\frac{dy}{dx} = -\frac{1}{x^2 + y^2}, \quad y(0) = 0$$

by using Modified Euler's method and obtain $y(0.3)$. Consider $h = 0.1$.

(c) To solve the boundary value problem

$$\frac{d^2y}{dx^2} = 4 \frac{dy}{dx} + 2y = 2 \sin x$$

$y(0) = y(1.0) = 1.0$, $y(1.0) = 1.0$

in the interval $[0, 1]$ with $h = 0.25$ by using finite difference method, a linear
system of equations in unknowns $y_1, y_2$ and $y_3$ is formed. Determine this linear
system of equations.
2015-16  
B.TECH. (AUTUMN SEMESTER) EXAMINATION  
ELECTRONICS ENGINEERING  
ELECTRONIC DEVICES  
EE-211.N  

Maximum Marks: 60  
Credits: 04  
Duration: Three Hours  

Answer all the questions. Assume suitable data if missing. Notations and symbols used have their usual meaning.

Values of some constants: Energy gap for Si = 1.12 eV; Thermal voltage (kT/e) = 25 mV at room Temp.; Permittivity of free space (ε₀ = 8.85 × 10⁻¹² F/m); Permittivity of silicon (ε_sil = 12 ε₀); Permittivity of oxide (ε_ox = 3.9 ε₀); μ_e = 400 cm²/V.s; μ_h = 800 cm²/V.s; n_i = 1 × 10¹⁶ cm⁻³; N_p = 1.04 × 10¹⁹ cm⁻³; N_n = 2.8 × 10¹⁷ cm⁻³.

<table>
<thead>
<tr>
<th>Q.No.</th>
<th>Questions</th>
<th>MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(a)</td>
<td>Explain the mobility and how it is related with diffusion constant.</td>
<td>02</td>
</tr>
<tr>
<td>1(b)</td>
<td>Explain Hall Effect and its application.</td>
<td>02</td>
</tr>
<tr>
<td>1(c)</td>
<td>Define drift current and obtain an expression for conductivity.</td>
<td>02</td>
</tr>
<tr>
<td>1(d)</td>
<td>What is a linearly graded pn junction? Derive the expression of diode equation for an abrupt pn junction.</td>
<td>06</td>
</tr>
<tr>
<td>1(e)</td>
<td>The electron and hole mobilities of Si sample are μ_e = 800 cm²/V.s and μ_h = 400 cm²/V.s respectively. If for silicon sample, n_i = 1 × 10¹⁶ cm⁻³, Determine the conductivity of intrinsic Si at room Temp. The sample is then doped with N_D = 2.8 × 10¹⁷ cm⁻³ phosphorous atoms. Determine the equilibrium hole concentration and conductivity of doped sample.</td>
<td>03</td>
</tr>
</tbody>
</table>

OR

1'(a) An abrupt silicon p-n junction consists of a p-type region containing 10¹⁶ cm⁻³ acceptors and an n-type region containing 5 × 10¹⁶ cm⁻³ donors.  
   i. Calculate the built-in potential of this p-n junction.  
   ii. Calculate the total width of the depletion region if the applied voltage V is 0, 0.5, and -2.5 V.  
   iii. Calculate maximum electric field in the depletion region at 0, 0.5, and -2.5 V.  

1'(b) What is a Varactor Diode? Show that m = 3/2 yield the desired capacitance characteristic.  

1'(c) Determine output waveform for the network of Figure 1 for the input indicated.  

Contd......2.
2(a) What are the small signal models of BJT? Explain with the help of diagrams. Also, derive the expression of the transconductance.  

2(b) For BJT's shown in Figure 2: $R_o = 1K\Omega$, $R_e = 10K\Omega$, $R_{se} = 100\Omega$, $J_e = 1.5mA$, $V_{mm} = 2.5mV$ a $\beta = 99$ and $r_o = \infty$. Determine transconductance, input impedance and output impedance for each BJT circuit.

2(c) How the internal capacitance in BJT affect the performance of BJT amplifiers? Derive the expression for unity gain bandwidth ($\omega_T$).

$OR$

2'(c) When the CE amplifier shown in Figure 3 is biased with a certain $V_{be}$, the dc voltage at the collector is found to be -2 V. For $V_{CC} = 5V$ and $R_C = 1k\Omega$, find $I_C$ and small signal voltage gain. For a change $\Delta V_{be} = 5mV$, calculate the resulting $\Delta V_C$. Repeat for $\Delta V_{be} = -5mV$. 

Contd....3.
3(a) The NMOS and PMOS transistors in the circuit shown in Figure 4 below, are matched with $k_n'(W_0/L_0) - k_p'(W_p/L_p) = 1mA/V^2$ and $V_{th} = V_{tp} = 1V$. Assuming $\lambda = 0$ for both devices, find the drain currents $i_{DS}$ and $i_{DS}$ and the voltage $V_{ds}$ for $v_{in} = 0V$, $-2.5V$, and $-2.5V$. Given $V_{DD} = V_{SS} = 2.5V$.

3(b) What are the different parasitic capacitances of MOSFET? What are their values for different region of operation?

3(c) For the NMOS amplifier shown in Figure 5 below, derive the expressions for the voltage gains $v_{o}/v_{i}$ and $v_{o}/v_{i}$. Also find the input impedance.
3(a) A discrete MOSFET common-source amplifier has \( R_{in} = 2 \text{M}\Omega \), \( g_m = 4 \text{mA/V} \), \( r_o = 100 \text{k}\Omega \), \( R_C = 10 \text{k}\Omega \), \( C_{ps} = 2 \text{pF} \), and \( C_{gs} = 0.5 \text{pF} \). The amplifier is fed from a voltage source with an internal resistance of 500 \( \text{k}\Omega \) and is connected to a 10-k\Omega load. Find:
(a) The overall midband gain \( A_M \)
(b) the upper 3-dB frequency \( f_H \)

4(a) What is amount of feedback? Show that the input impedance of voltage amplifier and output impedance of current amplifier increases with negative feedback.

4(b) The noninverting buffer op-amp shown in Figure 6. Assuming that the op-amp has infinite input resistance, what is \( \beta \)? If \( A = 100 \), what is closed loop voltage gain? What is amount of feedback (in dB)? For Voltage Signal Source of 1 V, find output voltage and input voltage. If \( A \) decreases by 10\%, what is the corresponding decrease in gain with feedback.

![Figure 6](image)

4(e) In a particular amplifier design, the \( \beta \) network consists of a linear potentiometer for which \( \beta \) is zero at one end, 1 at the other end and 0.50 in the middle. As the potentiometer is adjusted, find the three values of closed loop gain (at three positions of potentiometer) that result when the amplifier open loop gain is (I) 1 (II) 10 and (III) 100.

4(d) Draw the diagram of Wein's bridge oscillator and derive the expression for condition of oscillation as well as frequency of oscillation.

OR

4'(d) Draw the diagram of RC phase shift oscillator and derive the expression for condition of oscillation as well as frequency of oscillation.
Question 1(a):
If the voltage across a 2-H inductor is known to be $10\cos3t\, V$, what information is then available about the inductor current? Moreover, completely specify the current if it is also known that the current is 1 A at $t = -\pi/2\, s$.

OR

Question 1(a'):
The switch in the circuit in Fig. 1 has been closed for a long time, and is opened at $t = 0$. Find $v(t)$ for $t \geq 0$. Calculate the initial energy stored in the capacitor.

![Circuit Diagram](image)

Question 1(b):
For what type of circuits are commonly used theorems like Superposition etc. valid? Give reasons. Also explain how the inductor and capacitor are linear elements, even though the voltage and current are having a differential or integral relationship.

Question 2(a):
With the help of a network state and verify Millman's and Thevenin's theorems.

Question 2(b):
Determine the current in 20 $\Omega$ resistor of the network shown in Fig. 2 by Thevenin's theorem.
2(b') Determine the mesh currents for the network shown in Fig. 3.

3(a) Prove the statement that "In a linear graph every cut set has an even number of branches in common with every loop".

3(b) With the help of a detailed graph, determine the relation between
- branch current and loop current
- branch voltage and node voltage

OR

3'(b) With the help of a detailed graph, explain:
- Kirchhoff's Current Law for the fundamental cut-sets
- Kirchhoff's Voltage Law for the fundamental loops

4(a) Give the salient features of state variable analysis.

4(b) Write the normal-form state variable equations, through a step-wise procedure for the circuits given in Fig. 4.

Contd.....3.
4(b') Give the state variable representation of the circuits given in Fig. 5. What is the order of these circuits and why? [08]

5(a) Obtain the general equivalent circuit for a two-port if its z-parameters are given. Also find the T-equivalent for the special case of a reciprocal network. [06]

5(b) Obtain the overall parameters of a series and parallel connection of two-port networks. [06]
2015-16
B.TECH. (AUTUMN SEMESTER) EXAMINATION
ELECTRONICS ENGINEERING
ELECTRONIC INSTRUMENTATION
EL-222

Maximum Marks: 60
Credits: 04
Duration: Three Hours

Answer all the questions.
Assume suitable data if missing.
Notations and symbols used have their usual meaning.

Q.No. | Question | M.M.
--- | --- | ---
1(a) | Explain with suitable diagram, the construction and the theory of operation of an attraction type OR a repulsion type moving iron instrument. | [06]
1(b) | The coil of a 150 V MI voltmeter has a resistance of 400Ω and an inductance of 0.75 H. The current drawn by the instrument when placed on a 150 V d.c. supply is 0.05 A. Calculate (i) the alteration of the reading between the d.c. and a.c. at 100 Hz, (ii) the capacitance required to eliminate the frequency error. | [06]

2(a) | Explain the construction and principle of operation of an induction type wattmeter OR Hall effect wattmeter. | [06]
2(b) | A dynamometer type wattmeter connected normally to read power indicates the value \( P_1 \). A second reading \( P_2 \) is obtained when a capacitor of reactance equal to the pressure coil resistance is connected in series with the pressure coil. Show that the phase angle \( \phi \) of the load can be obtained from the expression, \( \tan \phi = 1 - \frac{2P_2}{P_1} \). | [06]

3(a) | Draw the Schering's OR Wein's bridge for the measurement of capacitance and hence, derive its balance condition relations. | [06]
3(b) | A coil with a resistance of 3Ω is connected to the terminals of the Q-meter. Resonance occurs at an oscillator frequency of 5 MHz and a resonating capacitance of 100 pF. Find the percentage error introduced by the insertion resistance of 0.1Ω. | [06]

4(a) | Name the special purpose CROs and explain them briefly. Mention their applications also. | [06]
4(b) | What are the common measurement applications of CROs? Explain them briefly. | [06]

5(a) | Describe with neat sketches any two types of transducers for the measurement of temperature OR pressure. | [06]
5(b) | A thermistor has a resistance of 4kΩ at 0°C and 300Ω at 40°C. Determine the range of resistance to be measured if the temperature rises from 50°C to 100°C. | [06]
1(a) Determine whether or not each of the following signals is periodic. If the signal is periodic, determine its fundamental period.

(i) \( x(t) = E \nu[\cos(4 \pi t) u(t)] \)

(ii) \( x[n] = \cos\left(\frac{n}{2} \pi\right) \cos\left(\frac{\pi}{4} n\right) \)

OR

1(a') Determine whether the following signals are power or energy signals or neither.

(i) \( x(t) = t u(t) \)

(ii) \( x[n] = e^{-j\left(\frac{\pi}{2} n + \frac{\pi}{6}\right)} \)

1(b) Consider the discrete-time signal \( x[n] = 1 - \sum_{k=-3}^{0} \delta(n - 1 - k) \). Determine the values of the integers \( M \) and \( n_0 \) so that \( x[n] \) may be expressed as \( x[n] = u[Mn - n_0] \).

1(c) Consider a continuous-time system with input \( x(t) \) and output \( y(t) \) related by \( y(t) = x(\sin(t)) \). Determine the causality and linearity properties of the system.

OR

1(c') Consider a discrete-time system with input \( x[n] \) and output \( y[n] \) related by \( y[n] = x[n^2] \). Determine whether it is (i) memory-less (ii) stable (iii) time-invariant (iv) causal.

1(d) A continuous time periodic signal \( x(t) \) is real-valued and has a fundamental period \( T = 8 \). The non-zero Fourier series coefficients of \( x(t) \) are:
\[
X_1 = X_{-1} = 2, \quad X_3 = X_{-3} = 4j
\]

Express \( x(t) \) in the form \( x(t) = \sum_{n=0}^{\infty} A_n \cos(\omega_n t + \phi_n). \)

1(e) Evaluate \( y[n] = x[n] * h[n] \) where \( x[n] = \begin{cases} 1, & 0 \leq n \leq 3 \\ 0, & \text{otherwise} \end{cases} \) and \( h[n] = \begin{cases} 1, & 0 \leq n \leq 2 \\ 0, & \text{otherwise} \end{cases} \) by using graphical method.

OR

1(e') For \( x(t) \) and \( y(t) \) shown in Figure 1, evaluate \( y(t) = x(t) * h(t). \)

![Figure 1](image)

2(a) Consider an exponentially damped sinusoidal signal defined by \( g(t) = e^{-at} \cos(2\pi f_o t) u(t). \) Find the Fourier transform of \( g(t). \)

2(b) If the Nyquist rate for \( x(t) \) is \( f_x. \) Find the Nyquist rate for each of the following signals:

(i) \( x(t) \cos(\omega_0 t) \)
(ii) \( x(2t) \)
(iii) \( x(t) * x(t) \)

2(b) Find the inverse Laplace transform of \( X(s) = \frac{2s^2 - s - 3}{(s+1)(s-1)(s+2)} \), if the region of convergence is: i) \( \text{Re}(s) > 1 \) ii) \( -1 < \text{Re}(s) < 1 \) iii) \( -2 < \text{Re}(s) < -1 \)

Also comment on causality and stability of the system in each case.

2(c) Compute the convolution of the signals \( x_1[n] \) and \( x_2[n] \) given below, using \( z \)-transform.

\[
x_1[n] = \begin{cases} 1, & n = -2, 1 \\ 0, & \text{otherwise} \end{cases}
\]

\[
x_2[n] = \begin{cases} 1, & 0 \leq n \leq 5 \\ 0, & \text{otherwise} \end{cases}
\]

OR

2(a') Let \( x(t) \) be a signal with Fourier transform \( X(f). \) Suppose you are given the following facts:

\[
\text{Cont... 3}
\]
(i) \( x(t) \) is real.
(ii) \( x(t) = 0 \) for \( t < 0 \).
(iii) \( \int_{-\infty}^{\infty} e \{ X(f) \} e^{i2\pi ft} \, df = |t| e^{-|t|} \).

Determine the closed form expression for \( x(t) \).

2'(b) Compute the Hilbert transform and pre-envelope of \( x(t) = \sin(2t) \).

2'(c) Evaluate the inverse z-transform of \( X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}} \). If the region of convergence is:
   i) ROC \( |z| > 1 \)
   ii) ROC \( 0.5 < |z| < 1 \). Also comment on causality and stability of the system in each case.

2'(d) Differentiate clearly between DTFT and DFT.

3(a) Obtain the closed loop transfer function \( \frac{C(s)}{R(s)} \) for the system’s block diagram as shown in Figure 2, using:
   (i) Block Diagram reduction technique
   (ii) Mason’s Gain Formula.

\[ \text{Figure 2} \]

3(b) The open loop transfer function of a unity-feedback system is \( G(s) = \frac{1}{s(s+1)} \).

Determine the nature of the response of the closed loop system for unit step input.
Also, determine the rise time, peak time and peak overshoot.

4(a) Let two honest coins, marked 1 and 2, be tossed together. The four possible
outcomes are $T_1T_2$, $T_1H_2$, $H_1T_2$, $H_1H_2$. ($T_1$ indicates toss of coin 1 resulting in tails; similarly $T_2$ etc.) We shall treat that all these outcomes are equally likely; that is, the probability of occurrence of any of these four outcomes is $\frac{1}{4}$. (Treating each of these outcomes as an event, we find that these events are mutually exclusive and exhaustive). Let the event $A$ be 'not $H_1H_2$' and $B$ be the event 'match'. (Match comprises the two outcomes $T_1T_2$, $H_1H_2$). Find $P(B|A)$. Are $A$ and $B$ independent?

OR

4(a*) Consider a random variable $X$ having probability density function

$$f_X(x) = \begin{cases} 
1 - \frac{x}{2}, & 0 \leq x \leq 2 \\
0, & \text{otherwise}
\end{cases}$$

Let $Y = bX$, where $b$ is a constant. Find $f_Y(y)$.

4(b) Consider a random process $X(t) = A \cos(2\pi f_c t + \theta)$, where the phase $\theta$ is a uniformly distributed random variable over the range $0$ to $2\pi$. Find and plot the autocorrelation function (ACF) and power spectral density of this random process.

4(c) Define power spectral density (PSD) of a wide-sense stationary (WSS) random process and prove the following properties of PSD:

(i) PSD at zero frequency is the area under the autocorrelation function.

(ii) The power spectral density of a real-valued random process is an even function of frequency.

(iii) PSD of a WSS random process is always positive.
2015-2016

B.TECH. AUTUMN (III SEMESTER) EXAMINATION
(ELECTRONICS ENGINEERING)
HIGHER MATHEMATICS - I
AM 251
Credits-04

Maximum Marks: 60

Duration: Two Hours

Note:- 1. Answer all the questions.
2. Standard Normal Table is attached (SHEET NO: 3)

Q1. (a) (i) Evaluate the Laplace transform of \( f(t) = e^{-4u} \int_0^t \left( \frac{\sin 3u}{u} \right) du. \) \[ 8 \]

(ii) Evaluate the inverse Laplace transform of \( \tilde{f}(s) = s \ln \frac{s-1}{s-3} + 7. \)

OR (a') (i) Express the function \( f(t) \) in terms of unit step function and hence obtain its Laplace transform where

\[ f(t) = \begin{cases} t^2 & \text{for } 0 < t \leq 2 \\ 4t & \text{for } t > 2 \end{cases} \]

(ii) Solve, by using convolution theorem, the integral equation

\[ f(t) + \int_0^t f(\lambda) \cos(t - \lambda) d\lambda = e^{-t} \]

(b) Solve by Laplace transform method the initial value problem:

\( (D^2 + 4D + 13)x = e^{-t} \)

given that \( x = 0, \ Dx = 2 \) at \( t = 0, \) where \( D \equiv \frac{d}{dt}. \)

Q2 (a) Find the values of \( a, b, c \) such that the maximum value of directional derivative of \( \phi = axy^2 + byz + cz^2x^2 \) at the point \( (1, -1, 1) \) is in the direction parallel to the axis of \( y \) and has magnitude 6.

OR (a') Show that \( \text{div}(\text{grad} \ r^n) = n(n+1)r^{n-2} \) where \( r \) is in its usual sense.

(b) Show that there exists a scalar function \( \phi \) such that \( \vec{A} = \text{grad} \phi \)

for \( \vec{A} = \frac{\vec{r}}{r^5} \) where \( \vec{r} \) and \( r \) have usual meaning. Also find \( \phi \) given that \( \phi(1) = 0. \)
Q3. (a) Using Green's Theorem or otherwise find the work by the force \( \mathbf{F} = (x^2 - y^3)i + (x + y)j \) in moving a particle along the closed path \( C \) containing the curve \( x + y = 0, x^2 + y^2 = 16, y = x \) in the first and fourth quadrants.

(b) Use divergence theorem to evaluate \( \iint_S \mathbf{A} \cdot \mathbf{n} \, dS \) where

\[
\mathbf{A} = x^2z \mathbf{i} + yj - xz^2k
\]

and \( S \) is the boundary of the region bounded by the paraboloid \( z = x^2 + y^2 \) and the plane \( z = 4y \).

OR

(b') Verify Stokes's Theorem for the function

\[
\mathbf{A} = (2x - y)i - yz^2j - y^2zk
\]

where the open surface is the upper half of the sphere \( x^2 + y^2 + z^2 = 1 \).

Q4. (a) If \( A \) and \( B \) are independent events associated to some experiment \( E \), prove that \( A \) and \( B^c \) are also independent. Also, if a random variable \( X \) assumes four values with probabilities

\[
\frac{1 + 3x}{4}, \frac{1 - x}{4}, \frac{1 + 2x}{4}, \frac{1 - 4x}{4}
\]

for what values of \( x \) is this a probability distribution?

OR

(a') Suppose that \( A, B, \) and \( C \) are events such that \( P(A) = P(B) = P(C) = \frac{1}{4}, P(A \cap B) = P(B \cap C) = 0 \) and \( P(A \cap C) = \frac{1}{8} \). Evaluate the probability that at least one of the events \( A, B, \) or \( C \) occurs.

Also, the diameter of an electric cable is normally distributed with mean 0.8 and variance 0.0004. The cable is considered defective if the diameter differs from its mean by more than 0.025. What is the probability of obtaining a defective cable?

(b) Suppose that the life length (in hours) of certain radio tube is a continuous random variable \( X \) with pdf

\[
f(x) = \begin{cases} 
\frac{\lambda}{x^2}, & x > 100 \\
0, & \text{elsewhere}.
\end{cases}
\]

(i) For what value of \( \lambda \), \( f \) is a legitimate probability density function?

(ii) What is the probability that a tube will last less than 200 hours if it is known that the tube is still functioning after 150 hours of service?

(iii) What is the probability that if 3 such tubes are installed in a set, exactly one has to be replaced after 150 hours of service?
### Standard Normal Probabilities

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