2015-2016

B.E. (III Semester) Examination
(Mechanical)
Higher Mathematics
[EAM - 231]
Credits - 04

Max Marks: 60

Duration: Three Hours

Note: Answer all questions. Programmable calculator is not allowed.

1. (a) Evaluate
(i) \[ \mathbb{L}[t e^{-t} \cosh t] \]
(ii) A function \( f(t) \) satisfies the equation \( f(t) + 2 \int_0^t f(t) \, dt = \cosh 2t \). Find the Laplace transform of \( f(t) \).

(b) Find inverse Laplace transform of
(i) \( \log \left( \frac{s + a}{s + b} \right) \),
(ii) \( \frac{s - 1}{s^2 - 6s + 25} \)

(c) Using Laplace transformation method, find the solution of the boundary value problem:
\[ \frac{d^2x}{dt^2} + 9x = \cos 2t, \text{ if } x(0) = 1, x \left( \frac{\pi}{2} \right) = -1. \]

OR

(c') Using Laplace transformation method solve the equation
\[ 2 \frac{d^2y}{dt^2} + 5 \frac{dy}{dt} + 2y = e^{-2t}, \quad y(0) = 1, y'(0) = 1. \]

[5+5+5]

2. (a) Show that \( \text{div} (\text{grad} \, r^n) = n (n+1) \, r^{n-2} \).

(b) Find the directional derivative of \( \phi = e^{2x} \cos yz \) at the origin in the direction of the tangent to the curve \( x = a \sin t, y = a \cos t, z = at = \frac{\pi}{4} \).

(c) Give the vector \( \vec{F} = (x^2 - y^2 + 2xz) \hat{i} + (xz - xy + yz) \hat{j} + (z^2 + x^2) \hat{k} \). Find \( \text{curl} \, \vec{F} \). Show that the vector given by \( \text{curl} \, \vec{F} \) at \( P_0 (1, 2, -3) \) and \( P_1 (2, 3, 12) \) are orthogonal.

OR

(c') Show that vector function \( \vec{A} = (y \sin z - \sin x) \hat{i} + (x \sin z + 2yz) \hat{j} + (xy \cos z + y^2) \hat{k} \)
is irrotational. Find its scalar potential.

[5+5+5]

Contd......2
3. (a) The acceleration of a particle at time $t$ is given by $\ddot{a} = 18 \cos 3t \mathbf{i} - 8 \sin 2t \mathbf{j} + 6t \mathbf{k}$. If velocity $\mathbf{v}$ and displacement $\mathbf{r}$ be zero at $t = 0$, find $\mathbf{v}$ and $\mathbf{r}$ at any time $t$.

(b) Verify Green's theorem to evaluate the line integral $\int_C (2y^2 dx + 3xdy)$, where $C$ is the boundary of the closed region bounded by the curves $y = x$ and $y = x^2$.

OR

(b') Verify the divergence theorem for $\mathbf{F} = 4xz \mathbf{i} - y \mathbf{j} + yz \mathbf{k}$ taken over the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

(c) If $\mathbf{F} = y \mathbf{i} + (x - 2xz) \mathbf{j} - xy \mathbf{k}$, evaluate, using Stokes' theorem, $\iint_S \text{curl} \mathbf{F} \cdot \mathbf{n} \, dS$, where $S$ is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane.

4. (a) Show that the polar form of Cauchy – Riemann equation is

$$\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \quad \frac{\partial u}{\partial \theta} = -\frac{1}{r} \frac{\partial u}{\partial r}.$$  

Use the result to show that $\log z$ is analytic function.

(b) If $f(z) = u + iv$ is analytic function and $u - v = (x - y)(x^2 + 4xy + y^2)$, find $f(z)$ in terms of $z$.

(c) Using Cauchy's integral theorem find the value of the integral $\int_C \frac{z + 4}{z^2 + 2z + 5} \, dz$, where $C$ is the circle $|z + 1| = 1$.

OR

(c') Use Cauchy's integral formula to evaluate $\int_C \frac{2z + 1}{z^2 + z} \, dz$, where $C$ is $|z| = \frac{1}{2}$.
Answer all questions. Assume suitable data if missing. Notations used have their usual meaning.

Q.No. Question
1(a) What do you understand by state of stress at a point? Prove that stress tensor \( \tau_{ij} = \tau_{ji} \)
1(b) The stresses at a point are given as \( \sigma_{x} = 200 \text{ MPa}, \tau_{xy} = 100 \text{ MPa} \) and \( \sigma_{yy} = 0 \) with reference to \( x, y \) axes. If the new set of axes \( x', y' \) is formed by rotating \( xy \) through 60° about \( z \) axis. Find \( \sigma_{x'x'}, \tau_{x'y'} \) and \( \sigma_{y'y'} \).

OR

1'(b) What do you mean by strain rosettes? Describe different types of strain rosettes. The data for a delta rosette are given as follows:
\[ \epsilon_{00} = 200 \times 10^{-6}, \epsilon_{600} = 200 \times 10^{-6}, \epsilon_{1200} = -300 \times 10^{-6} \]
Find the principle strain and maximum shear stress and their directions.

2. (a) Write the assumptions in which the Euler's formula for buckling load is based. Also write the limitations of this formula.

2. (b) The external and internal diameter of a hollow cast iron column are 5 cm and 4 cm respectively. If the length of the column is 3 m and both of its ends are fixed. Determine crippling load using Rankine's formula.
Take \( f_{c} = 550 \text{ N/mm}^{2} \) and constant \( a = 1/1600 \) in Rankine's Formula.

OR

2'(b) What do you mean by strain energy? Differentiate between elastic strain energy and total strain energy. Derive the formula for maximum strain energy stored when a beam of constant rectangular section \((b \times d)\) and the length \( L \) is subjected to a shear force \( F \).

3. (a) Determine the maximum radial and circumferential stresses in a flat steel disc 60 cms in diameter and of uniform thickness rotating at 2940 rpm, \( \nu = 0.25 \). Also (i) Solve the above problem if there is a pin hole at the centre. (ii) Solve the above problem if there is a hole of 20 cm diameter at the centre.

3. (b) Explain Generalized Hooke's law for isotropic materials. Derive the relationship between Young's Modulus and Shear Modulus for an isotropic material.

OR

Contd.....2.
3. (b) What are compatibility equations? Derive the following
\[
\frac{\partial}{\partial z} \left( \frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} \right) = 2 \frac{\partial^2 \varepsilon_{zz}}{\partial x \partial y}
\]

4. (a) A steel cylinder 1 m long, of 150 mm internal diameter and plate thickness 5 mm, is subjected to an internal pressure of 7 MPa; the increase in volume owing to the pressure is \(16.8 \times 10^{-5}\) m\(^3\). Find the values of Poisson's ratio and the modulus of rigidity. Assume \(E = 210\) GPa.

4. (b) What are wire wound thin cylinders? Derive its stress strain relations?

**OR**

4. (b') A thick cylinder of internal and external radii 300 mm and 500 mm respectively is subjected to a gradually increasing internal pressure \(P\). Determine the value of \(P\) when:
(a) the material of the cylinder first commences to yield
(b) yielding has progressed to mid-depth of the cylinder wall
(c) The cylinder material suffers complete "collapse".
Take \(\sigma_y = 600\) MPa.

5. (a) State and explain any two of the following
(i) Superposition Principle.
(ii) Macaulay's method.
(iii) Lame's Theory
(iv) Three Moment Theorem

5. (b) For the uniform beam and loading shown in Figure 1, determine the reaction at each support and the slope at end \(A\).

![Figure 1]

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(06) (06) (06) (04) (08)
Maximum Marks: 60  
Credits: 04  
Duration: Three Hours

Answer all the questions
Assume suitable data if missing
Notations used have their usual meaning.

1(a) Give step by step procedure of expend enable pattern casting. Write advantages and disadvantages of this casting method. [06]

1(b) Name various defects in casting. Explain how they can be removed. [06]

OR

1'(b) Figure 1 shows a typical bottom gating system. Derive the formula of filling time of mould based on the principle of fluid flow. [06]

Figure 1: Bottom gating.

2(a) Describe the types of flames obtained in Oxy-acetylene gas welding process. [06]

2(b) Differentiate between soldering and brazing. Explain briefly how brazing is carried out. [06]

OR

Contd....2.
2(b) Explain Gas Metal Arc Welding (GMAW) and Submerged Arc Welding (SAW) process.

3(a) What is hot and cold working process? Give the advantages and disadvantages of each process.

3(b) Derive the equation of bite angle in rolling process.

OR

3'(a) Determine the die and punch sizes for blanking a circular disc of 25 mm diameter from a C20 steel whose thickness is 1.8 mm.

Clearance \( C \) is given by \( C = 0.0032 \times t \times \sqrt{\tau} \)

where \( t \) is thickness of sheet, \( \tau = 294 \text{ MPa} \)

3'(b) Describe any two of the following:

(i) Punching

(ii) Deep drawing

(iii) Spinning

4(a) Describe various steps involved in powder metallurgy process.

4(b) Explain the difference between chemical machining and electrochemical machining.

OR

4'(b) Describe the working principle of electron-beam machining with schematic diagram.

5(a) Describe any two of the following

(iv) Extrusion process of plastics

(v) Blow moulding

(vi) Rotational moulding

5(b) Write a short note on rapid prototyping operations including different classifications and applications?
Q.No.

1(a). One bag contains 4 white balls and 3 black balls, and a second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?

1(b). In an industrial process, the diameter of a ball bearing is an important measurement. The buyer sets specifications for the diameter to be $3.0 \pm 0.01$ cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean $\mu = 3.0$ and standard deviation $\sigma = 0.005$. On average, how many manufactured ball bearings will be scrapped?

1(c). Find the following using statistical tables of Normal, Binomial and Poisson distribution respectively:

i. $P(z > 1.73)$

ii. $b(2; 10, 0.3)$

iii. $B(3; 15, 0.2)$

iv. $P(1; 5)$

OR

1'(a). If the probability that a fluorescent light has a useful life of at least 800 hours is 0.9, find the probabilities that among 20 such lights

(a) exactly 18 will have a useful life of at least 800 hours;

(c) at least 2 will not have a useful life of at least 800 hours.

1'(b). An important factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production data in the past, it has been determined that the particle size (in micrometers) distribution is characterized by

$$f(x) = \begin{cases} 3x^{-4}, & x > 1, \\ 0, & \text{elsewhere}. \end{cases}$$

(i) Verify that this is a valid density function.

(ii) Evaluate $F(x)$.
(iii) What is the probability that a random particle from the manufactured fuel exceeds 4 micrometers?

1'(c). Find the following using statistical tables of Normal, Binomial and Poisson distributions respectively:

i. \( P(z < -1.73) \)

ii. \( B(2; 15, 0.3) \)

iii. \( b(3; 12, 0.3) \)

iv. \( P(2; 5) \)

2(a). The amount of time that a drive-through bank teller spends on a customer is a random variable with a mean \( \mu = 3.2 \) minutes and a standard deviation \( \sigma = 1.6 \) minutes. If a random sample of 64 customers is observed, find the probability that their mean time at the teller’s window is

i. at most 2.7 minutes;

ii. at least 3.2 minutes but less than 3.4 minutes.

OR

2(a'). The amount of time that a customer spends waiting at an airport check-in counter is a random variable with mean 8.2 minutes and standard deviation 1.5 minutes. Suppose that a random sample of \( n = 49 \) customers is observed. Find the probability that the average time waiting in line for these customers is

i. Less than 10 minutes

ii. Between 5 and 10 minutes

2(b). A manufacturer of sports equipment has developed a new synthetic fishing line that the company claims has a mean breaking strength of 8 kilograms with a standard deviation of 0.5 kilogram. Test the hypothesis that \( \mu = 8 \) kilograms against the alternative that \( \mu < 8 \) kilograms if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kilograms. Use a 0.05 level of significance.

OR

2(b'). Test the hypothesis that the average content of containers of a particular lubricant is 10 litres if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 10.4, 10.3 and 9.8 litres. Use a 0.01 level of significance and assume that the distribution of contents is normal.

2(c). iii) Find the following from the tables of sampling distributions and also mention over the rough sketch of the respective distributions:

(a) \( \chi^2_{0.05} \) when \( v = 7 \)

(b) \( t_{0.05} \) when \( v = 13 \)

(c) \( F_{0.99}(9, 7) \)

(d) \( P(t > -2.567) \) when \( v = 11 \)

3. The following data were collected to determine the relationship between pressure and the corresponding scale reading for the purpose of calibration:
Pressure (x)  10  11  12  40  50
Scale reading (y)  13  16  20  86  92

(i) Find the estimate of regression line \( \mu_{y|x} = \beta_0 + \beta_1 x \)
(ii) Calculate the mean squared error and coefficient of determination.

4(a). Differentiate briefly between any one of the following instruments
(i) Contacting and Non-contacting type
(ii) Null and Deflection type

4(b). Explain any three of the following terms
(i) Resolution
(ii) Threshold
(iii) Static Sensitivity
(iv) Linearity
(v) Hysteresis
(vi) Back lash

4(c). Define dynamic characteristics of instrument systems. Derive the governing equation relating input and output signals for dynamic input for U-tube manometer.

OR

4(c'). A first order instrument is to measure signals with frequency content up to 100 Hz with an amplitude inaccuracy of 5%. What is the maximum allowable time constant? What will be the phase shift at 50 Hz?

5(a). Describe the working of Linear Variable Differential Transformer (L.VDT) type of transducer with the help of diagram.

OR

5(a'). Explain with the help of a neat sketch, the working of A-D Converters in detail.

6. Explain the working of the following measurement devices with neat sketch
   (a) Optical Pyrometer
   (b) Pirani Thermal Conductivity Gage
   (c) Rotameter
2015
B.E. (III SEMESTER) EXAMINATION
MECHANICAL
APPLIED THERMODYNAMICS
EME-221

Maximum Marks: 60 Credits: 04 Duration: Three Hours

Answer all the questions. Assume suitable data if missing. Notations used have their usual meaning.

<table>
<thead>
<tr>
<th>Q.No</th>
<th>Question</th>
<th>M.M.</th>
</tr>
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<tbody>
<tr>
<td>1(a)</td>
<td>Derive the general relation for the change in the internal energy of pure substance.</td>
<td>[6]</td>
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<tr>
<td>1(b)</td>
<td>Explain Joule Thomson coefficient? Describe the inversion curve on a temperature pressure diagram.</td>
<td>[6]</td>
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OR

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<td>1'(a)</td>
<td>Derive the Clapeyron equation.</td>
<td>[6]</td>
</tr>
<tr>
<td>1'(b)</td>
<td>Explain in detail the volume expansivity, and isothermal and adiabatic compressibility.</td>
<td>[6]</td>
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2(a) A water cooling tower for a power plant cools 45°C liquid water by evaporation. The tower receives air at 19.5°C and φ=30%, and 100 kPa that is blown through/over the water such that it leaves the tower at 25°C and φ=70%. The remaining liquid water flows back to the condenser at 30°C having given off 1MW, Find the mass flow rate of air, and determine the amount of water that evaporates. [6]

2(b) Consider a gas mixture that consists of 3 Kg of O₂ and 5 Kg of N₂, and 12 Kg of CH₄. Determine (a) the mass fraction of each component (b) the mole fraction of each component, and (c) the average molar mass and gas constant of the mixture. [6]

OR

2'(a) A rigid insulated vessel contains 12 Kg of oxygen at 200kPa and 280 K separated

Contd.....2.
by a membrane from a 26 Kg of carbon dioxide at 400 kPa and 360 K. The membrane is removed and the mixture comes to a uniform state. Find the final temperature and pressure of the mixture.

2'(b) An air-conditioning unit is shown in Fig., with pressure, temperature, and relative humidity data. Calculate the heat transfer per kilogram of dry air, assuming that changes in kinetic energy are negligible.

3(a) Explain the deviation of the actual vapour compression refrigeration cycle from the ideal cycle. Show various processes of the ideal and deviated cycle on T-s diagram.

3(b) A refrigerator has a steady flow of R-22 as saturated vapour at -20°C into the adiabatic compressor that brings it to 1000 kPa. After the compressor, the temperature is measured to be 60°C. Find the actual compressor work and the actual cycle coefficient of performance.

4(a) Briefly describe air standard cycle. What are the different air standard assumptions.

4(b) A gasoline engine has a volumetric compression ratio of 9. The state before compression is 290 K, 90 kPa, and the peak cycle temperature is 1800 K. Find the pressure after expansion, the cycle net work, and the cycle efficiency.

OR

4(b') A diesel engine has air before compression at 280 K and 85 kPa. The highest temperature is 2200 K, and the highest pressure is 6 MPa. Find the volumetric compression ratio and the mean effective pressure using cold air properties at 300 K.

Contd....3.
5(a) Out of the three compression processes (isothermal, adiabatic, polytropic), for the compressor, which requires the minimum work input. What are the different possible arrangements/techniques.

5(b) Write briefly on the classification of the compressor.

5(c) A single-stage, single-acting air compressor running at 1000 rev/min delivers air at 25 bar. For this purpose the induction and free air conditions can be taken as 1.013 bar and 15°C, and the FAD as 0.25m³/min. The clearance volume is 3% of the swept volume and the stroke/bore ratio is 1.2/1. Calculate.

(a) the bore and stroke

(b) the volumetric efficiency

(c) the indicated power

(d) the isothermal efficiency
1(a). For what uniform rotation rate in r/min about axis C will the U-tube in fig.1 take the configuration shown? The fluid is mercury at 20°C.

![Diagram of U-tube with dimensions: 20 cm, 12 cm, 10 cm, and 5 cm.]

1(b). A periodic Karman vortex street is formed when a uniform stream flows over a circular cylinder. Use the method of repeating variables to generate a dimensionless relationship for Karman vortex shedding frequency, $f_k$, as a function of free stream velocity $V$, fluid density $\rho$, fluid viscosity $\mu$, and cylinder diameter $D$.

2(a). Define the following with physical examples:
1. Streamline
2. Pathline
3. Streakline

2(b). An incompressible flow in polar coordinates is given by $v_r = K \cos \theta \left(1 - \frac{b}{r^2}\right)$ and $v_\theta = -K \sin \theta \left(1 + \frac{b}{r^2}\right)$. Does this field satisfy continuity? For consistency, what should be the dimensions of $K$ and $b$ be?
2(a). A two dimensional incompressible flow field is defined by

\[ u = \frac{-ky}{x^2+y^2}, \quad v = \frac{kx}{x^2+y^2} \]

Where \( K \) = constant. Is this flow irrotational? If so, find its velocity potential, sketch a few potential lines and interpret the flow pattern.

2(b). A general equation for a steady, two dimensional velocity field that is linear in both spatial direction (x and y) is given as \( \vec{V} = (U + a_2x + b_1y)\hat{i} + (V + a_2x + b_2y)\hat{j} \). Where U and V and the coefficients are constants. (i) What relationship must exist to ensure that the flow field is incompressible? (ii) Calculate the shear strain rate in xy-plane. (iii) Calculate the vorticity vector and its direction.

3(a). An incompressible fluid flows past an impermeable flat plate, as in Fig. 2, with a uniform inlet profile \( u = U_0 \) and a cubic polynomial exit profile \( u \approx U_0 \left( \frac{3y-\beta^3}{2\beta^3} \right) \), where \( \beta = \frac{y}{\delta} \). Compute the volume flow \( Q \) across the top surface of the control volume.

![Fig 2](image)

3(b). Water at 20°C flows through the elbow in Fig. 3, and exits to the atmosphere. The pipe diameter is \( D_1 = 10 \) cm, while \( D_2 = 3 \) cm. At a weight flow rate of 150 N/s, the pressure \( p_1 = 2.3 \) atm (gage). Neglecting the weight of water and elbow, estimate the force on the flange bolts at section-1.

![Fig 3](image)

3(a). Derive Bernoulli’s equation in Cartesian system of coordinates. State clearly the assumptions and its violation in different physical scenario.
3(b). A 20°C water jet strikes a vane mounted on a tank with frictionless wheels, as in Fig. 4. The jet turns and falls into the tank without spilling out. If \( \theta = 30^\circ \), evaluate the horizontal force \( F \) required to hold the tank stationary.

![Fig 4](image)

4(a). For two-dimensional, incompressible, irrotational flow, the superposition of a doublet and a uniform flow represents flow around a circular cylinder. Obtain the stream function and velocity potential for this flow pattern. Find the velocity field, locate the stagnation points and the cylinder surface, and obtain the surface pressure distribution. Integrate the pressure distribution to obtain the drag and lift forces on the circular cylinder.

![Fig 4](image)


5(a). Derive the equation for velocity for fully developed laminar flow in a pipe. Hence derive volume flow rate as a function of pressure drop \( \Delta p \), and show that maximum velocity is two times the average velocity.

5(b). Water at 20°C is to be siphoned through a tube 1 m long and 2 mm in diameter, as in Fig. 5. What is the flow rate if \( H = 50 \) cm? Is there any height \( H \) for which the flow might not be laminar? Neglect the tube curvature and take \( \mu_{\text{water}} = 0.001 \text{kg/m-s} \) and \( \rho_{\text{water}} = 998 \text{ kg/m}^3 \).

![Fig 5](image)