SYLLABUS FOR DESCRIPTIVE QUESTIONS

ALGEBRA

Group Theory:
Relation of conjugacy, Conjugate classes of a group, Number of elements in a conjugate class of an element of a finite group, Class equation in a finite group and related results, Partition of a positive integer, Conjugate classes in $S_n$, Sylow’s theorems, External and internal direct products and related results. Commutator subgroup of a group, Subnormal series of a group, Refinement of a subnormal series, Length of a subnormal series, Solvable groups and related results, Nilpotent groups, relation between solvable and nilpotent groups, Composition series of a group, Zassenhaus theorem, Schreier refinement theorem, Jordan-Holder theorem for finite groups.

Rings Theory and Modules:
Examples and Fundamental properties of rings and homomorphism, Quotient rings sums of ideals, Direct sum of ideals, prime ideals and m-systems, Semi prime ideals, The prime radical of a ring, Prime rings, D.C.C. and the prime radical, Ideals in complete matrix rings, Subdirectly irreducible rings, Boolean rings, Rings and modules or quotient, Extension and contraction of ideals, Local rings, Localization of a ring at a prime ideal, Properties of rings of quotients.
Modules and submodules, sum and intersection of submodules, Linear combinations and spanning set, Homomorphisms, Isomorphisms Theorems, Inverse image of submodules, Annihilator, Order of elements. Direct summands, Split homomorphisms, Projections, Idempotent endomorphisms, Essential and superfluous submodules, Semi-simple modules, socle and radical of modules, Basis and rank free modules.

Field Theory and Homological Algebra:
Extensions, Algebraic extensions, Splitting fields, Normal extensions, Multiple roots, Finite fields and separable extensions, Automorphism groups and fixed fields, Fundamental Theorems of Galois Theory, Fundamental Theorem of Algebra.
Direct products and direct sums of modules, Free modules, Annihilation, Chain conditions, Noetherian modules, Artinian modules, Exact sequences, Short exact sequences, Splitting sequences, The four Lemma, The five Lemma, Semi exactness, Tensor product of modules, Injective modules, Character modules of free modules, Connections to injectivity, Maximal essential extensions, Modules with composition series, Jordan Holder Theorem, Composition.

Linear Algebra:
Linear functional, Dual basis, Computing of a dual basis, Dual vector spaces, Annihilator, Second dual space, Dual transformations. Inner-product spaces, Normed space, Cauchy-Schwartz inequality, Pythagorean Theorem, Projections, Orthogonal Projections, Orthogonal complements, Orthonormality, Matrix Representation of Inner-products, Gram-Schmidt Orthonormalization Process, Bessel’s Inequality, Riesz Representation theorem and orthogonal Transformation, Inner product space isomorphism.
Operators on Inner-product spaces, Isometry on Inner-product spaces and related theorems, Adjoint operator, selfadjoint operator, normal operator and their properties, Matrix of adjoint operator, Algebra of Hom(V,V), Minimal Polynomial, Invertible Linear transformation, Characteristic Roots, Characteristic Polynomial and related results,
Diagonalization of Matrices, Invariant Subspaces, Cayley-Hamilton Theorem, Canonical form, Jordan Form, Forms on vector spaces, Bilinear Functionals, Symmetric Bilinear Forms, Skew Symmetric Bilinear Forms, Rank of Bilinear Forms, Quadratic Forms, Classification of Real Quadratic forms.

**ANALYSIS**

**Real Analysis:**
Sequences and series of functions, pointwise and uniform convergence, tests for uniform convergence: Cauchy criterion, Weierstrass’s M-test, Abel’s and Dirichlet’s test, uniform convergence and continuity, uniform convergence and differentiation, uniform convergence and integration, Weierstrass’s approximation theorem, existence and properties of Riemann-Stieljes integrals, integration of vector valued functions, rectifiable curves, power series, uniqueness theorem, Abel’s and Taylor’s theorems, rearrangement of terms of series and related results, functions of several variables, partial derivatives, derivatives on open set, chain rule, interchange of order of differentiation, inverse function theorem and implicit function theorem.

Measurable and non-measurable sets, measurable functions, Borel Lebesgue measureablity, measure and outer measure, extensions of a measure, uniqueness of extension, completion of measure, integration of non-negative functions, Riemann and Lebesgue integrals, the four derivatives, Lebesgue differentiation, differentiation and integration, measure spaces, convergence in measure.

**Complex Analysis:**
Curves in complex plane, properties of complex line integrals, fundamental theorem of line integrals (or contour integration), Cauchy’s theorem, Cauchy-Goursat theorem, symmetric, starlike, convex and simply connected domains, Cauchy’s theorem for a disk, Cauchy’s integral theorem, index of a closed curve, advanced versions of Cauchy integral formula with applications, Cauchy’s estimate, Morera’s theorem, Riemann’s removable singularity theorem.

Convergence of sequences and series of complex valued functions, Weierstrass’ M-test, power series as an analytic function, root and ratio test, uniqueness theorem for power series, zeros of analytic functions, identity theorem and related results, maximum/minimum modulus principles and theorems, Schwarz’ lemma and its consequences, Advanced versions of Liouville’s theorem, isolated and non-isolated singularities, removable singularities, poles, characterization of singularities through Laurent’s series.

Calculus of residues, residue at a finite point, results for computing residues, residue at the point at infinity, Cauchy’s residue theorem, residue formula, meromorphic functions, number of zeros and poles, argument principle, evaluation of integrals, Rouche’s theorem, Mittag-Leffler expansion theorem and their applications.

Conformal mappings, special types of transformations, basic properties of Möbius maps, images of circles and lines under Mobius maps, fixed points, characterizations of Möbius maps in terms of their fixed points, triples to triples under Möbius maps, cross-ratio and its invariance property, mappings of half-planes onto disks, Inverse function theorem and related results.

**Functional Analysis:**
Normed spaces, Banach spaces and their properties, open and closed spheres in normed spaces, denseness, separability and closedness, completion of normed linear spaces, Finite
dimensional normed spaces and subspaces, equivalent norms, compactness and finite
dimension, Riesz’s lemma, quotient spaces.
Bounded linear operators with their norms and properties, algebraic and topological dual
spaces, examples and properties of dual spaces, weak convergence and strong convergence,
reflexive normed spaces and different kinds of topologies, properties of reflexive normed
spaces.
Hahn-Banach theorems and their consequences (analytic and geometric forms), pointwise
and uniform boundedness, uniform boundedness principle and its applications, open and
closed maps, open mapping and closed graph theorems, their consequences and applications,
Banach contraction theorem with its applications.
Inner product space, parallelogram law, polarization identity and related results, Schwartz
inequality, Hilbert space, orthogonality of vectors, orthogonal complements, projection
theorem and related results, orthonormal sets and sequences, Bessel’s inequality, total
orthonormal sets, Parseval formula, separable Hilbert spaces, Riesz representation theorems
for bounded linear functional and bounded sesquilinear form, Hilbert adjoint operators, self
adjoint operators, unitary and normal operators.

ALLIED COURSES

Ordinary Differential Equations:
Introduction to initial value problem and boundary value problems, linear dependence and
independence of solutions, equations with constant as well as variable coefficients,
Wronskian, variation of parameter, method of undetermined coefficients, reduction of the
order of an equation, method of Laplace’s transform.
Lipchitz’s condition and Gron Wall’s inequality, Picards theorems, dependence of solution
on initial conditions and on the function, Continuation of solutions, Non-local existence of
solutions Systems as vector equations, existence and uniqueness of solutions.
Strum-Liouville’s system, Green’s function and its applications to boundary value problems,
some oscillation theorems such as Strum theorem, Strum comparison theorem and related
results.
System of first order matrix equation, fundamental matrix, Non-homogeneous linear system,
Linear system’s with constant as well as periodic coefficients.

Topology:
Definitions and examples of topological spaces, Topology induced by a metric, closed sets,
Closure, Dense subsets, Neighbourhoods, Interior, Exterior and boundary of a set
accumulation points and derived sets, Bases and subbases, Topology generated by the
subbasis, subspaces and relative topology, Continuous functions and homeomorphism.
First and second countable spaces, Lindelof spaces, Separable spaces, Second countability
and separability, Separation axioms, $T_0, T_1, T_2, T_3, T_4$ spaces and their characterizations and
basic properties, Urysohn’s lemma, Tietze extension theorem.
Compact spaces and their basic properties, Separation of a space, Connected spaces,
Connected sets in the real line, Totally disconnected spaces, Intermediate value theorem, path
connected, Components, Path components, Locally connected spaces, Locally path connected
spaces, Totally disconnected spaces, Continuous functions and connected sets.
Product topology (finite and infinite number of spaces), Tychonoff product topology in terms
of standard sub-base and its characterizations, Projection maps, Separation axioms and
product spaces, Connectedness and product spaces, Compactness and product spaces
(Tychonoff’s theorem), Countability and product spaces.
**Differential Geometry:**
Charts, Atlases, Manifolds, Differentiable structure on a manifold, Smooth maps, Tangent vectors and Tangent space.
Vector fields, Lie product of Vector fields, Jacobian of a smooth map, Integral curves on a manifold, One parameter group of transformation.
Connexion, parallelism, Geodesic, Covariant differentiation, Torsion, Curvature, Structure equation of Cartan, Bianchi identities.