Principle of Effective Stress

These slides have been prepared for B. Tech Students of Department of Civil Engineering, AMU, Aligarh

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Visualization of in-situ stresses

- When a load is applied to soil, it is carried by the water in the pores as well as the solid grains.
- The increase in pressure within the porewater causes **drainage** (flow out of the soil), and the load is transferred to the solid grains.
- The rate of drainage depends on the permeability of the soil.
- The strength and compressibility of the soil depend on the stresses within the solid granular fabric.
- These are called effective stresses, $\sigma'$

Effective Stress is computed by a simple equation:

$$\sigma' = \sigma - u$$

Where, $\sigma$ is the total stress and $u$ is the neutral stress
Concept of Effective stress

- The effective stress is NOT the contact stress between the soil solids.
- It is the average stress on a PLANE through the soil mass as shown.
Effective Stress...

The effective stress in a soil mass not subjected to external loads is computed from the unit weights of soil and water, and the depth of groundwater table.

Case-1

- Consider a soil element at a depth $z$ below the ground surface and the groundwater level (GWL) is at ground surface.
- The total vertical stress is:

$$\sigma = \gamma_{sat}z$$

The pore pressure is calculated as:

$$u = \gamma_wz$$

The effective stress is calculated as:

$$\sigma' = \sigma - u$$

Plugging the above equations, we get:

$$\sigma' = \gamma_{sat}z - \gamma_wz$$

$$\sigma' = z (\gamma_{sat} - \gamma_w)$$

$$\sigma' = \gamma'z$$
Effective Stress...

Case-2:
If water table is at a depth of \( z_w \) units below ground level, then

\[
\sigma = \gamma z_w + \gamma_{sat}(z - z_w)
\]

\[
u = (z - z_w)\gamma_w
\]

The effective stress is:

\[
\sigma' = \sigma - u = \gamma z_w + \gamma_{sat}(z - z_w) - (z - z_w)\gamma_w
\]

\[
\sigma' = \gamma z_w + \gamma'(z - z_w)
\]

In which,

\( \gamma' \) = submerged unit weight
\( \gamma_{sat} \) = saturated unit weight
\( \gamma_w \) = unit weight of water

This equation refers to the principle of effective stress and was first recognized by Terzaghi (1883–1963) in the mid-1920s during his research into soil consolidation.
Effective Stress...

• The principle of effective stress is the most important principle in soil mechanics.

• Deformation in soils is a function of effective stresses and NOT total stresses.

• The principle of effective stresses applies only to normal stresses and NOT to shear stresses.

• The porewater CANNOT sustain shear stresses and therefore the soil solids must resist the shear forces. Thus $\tau' = \tau$ where $\tau$ is the total shear stress and $\tau'$ is the effective shear stress.
Problem 1: Determine the total and effective vertical stresses and pore water pressure.
Plot their variation with depth for the soil profile shown below in Figure.
Within a soil layer, the unit weight is constant, and therefore the stresses vary linearly. Therefore, it is adequate if we compute the values at the layer interfaces and water table location, and join them by straight lines.

**Solution-1**

At the ground level,
\[ \sigma_v = 0 \; ; \; \sigma_v' = 0 \; ; \; \text{and} \; u = 0 \]

At 4 m depth,
\[ \sigma_v = (4)(17.8) = 71.2 \; \text{kPa}; \; u = 0 \]
\[ \therefore \sigma_v' = 71.2 \; \text{kPa} \]

At 6 m depth,
\[ \sigma_v = (4)(17.8) + (2)(18.5) = 108.2 \; \text{kPa} \]
\[ u = (2)(9.81) = 19.6 \; \text{kPa} \]
\[ \therefore \sigma_v' = 108.2 - 19.6 = 88.6 \; \text{kPa} \]

At 10 m depth,
\[ \sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) = 186.2 \; \text{kPa} \]
\[ u = (6)(9.81) = 58.9 \; \text{kPa} \]
\[ \therefore \sigma_v' = 186.2 - 58.9 = 127.3 \; \text{kPa} \]

At 15 m depth,
\[ \sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) + (5)(19.0) = 281.2 \; \text{kPa} \]
\[ u = (11)(9.81) = 107.9 \; \text{kPa} \]
\[ \therefore \sigma_v' = 281.2 - 107.9 = 173.3 \; \text{kPa} \]
Graphical Distribution

Gravely sand
\( \gamma_{\text{sat}} = 18.5 \text{kN/m}^3; \gamma_m = 17.8 \text{kN/m}^3 \)

Sand
\( \gamma_{\text{sat}} = 19.5 \text{kN/m}^3 \)

Sandy gravel
\( \gamma_{\text{sat}} = 19.0 \text{kN/m}^3 \)

Stress or Pressure (kPa)

Depth (m)
Problem-2

Calculate the effective stress for a soil element at depth 5 m in a uniform deposit of soil as shown in Figure below

Step-1: Calculate Unit weights

**Above GWT**

\[ \gamma = \left( \frac{G_s + Se}{1+e} \right) \]
\[ \gamma_w = \frac{G_s(1+w)}{1+e} \]
\[ e = \frac{wG_s}{S} \]

\[ e = \frac{0.3 \times 2.7}{0.6} = 1.35 \]

\[ = \frac{2.7(1 + 0.3)}{1 + 1.35} \times 9.8 = 14.6 \text{ kN/m}^3 \]

**Below GWT, soil is saturated, S = 1,**

\[ e = wG_s = 0.4 \times 2.7 = 1.08 \]

\[ \gamma_{sat} = \left( \frac{G_s + e}{1+e} \right) \gamma_w \]

\[ = \left( \frac{2.7 + 1.08}{1 + 1.08} \right) \times 9.8 = 17.8 \text{ kN/m}^3 \]
Step 2: Calculate effective stress,

Total Stress:

\[ \sigma_z = 2 \times \gamma + 3 \times \gamma_{sat} \]
\[ = 2 \times 14.6 + 3 \times 17.8 = 82.6 \text{kPa} \]

Porewater Pressure:

\[ u = 3 \times \gamma_w = 3 \times 9.8 = 29.4 \text{kPa} \]

Effective Stress:

\[ \sigma' = \sigma_z - u = 82.6 - 29.4 = 53.2 \text{kPa} \]

Alternatively,

Using Submerged Unit weight method:

\[ \sigma' = 2\gamma + 3(\gamma_{sat} - \gamma_w) = 2\gamma + 3\gamma' \]
\[ \sigma' = 2 \times 14.6 + 3(17.8 - 9.8) = 53.2 \text{kPa} \]
Problem 2: A borehole at a site depicts the soil profile as shown in Figure below. Plot the distribution of vertical total and effective stresses with depth.
Solution-3

Step 1: Calculate the unit weights.

0–2 m

\[ S = 40\% = 0.4; \quad w = 0.05 \]

\[ e = \frac{wG_s}{S} = \frac{0.05 \times 2.7}{0.4} = 0.34 \]

\[ \gamma = \frac{G_s(1 + w)}{1 + e} \quad \gamma_w = \frac{2.7(1 + 0.05)}{1 + 0.34} \quad 9.8 = 20.7 \text{ kN/m}^3 \]

2–5.4 m

\[ S = 1; \quad w = 0.12 \]

\[ e = wG_s = 0.12 \times 2.7 = 0.32 \]

\[ \gamma_{\text{sat}} = \left( \frac{G_s + e}{1 + e} \right) \gamma_w = \left( \frac{2.7 + 0.32}{1 + 0.32} \right) 9.8 = 22.4 \text{ kN/m}^3 \]

5.4–20.6 m

\[ S = 1; \quad w = 0.28 \]

\[ e = wG_s = 0.28 \times 2.7 = 0.76 \]

\[ \gamma_{\text{sat}} = \left( \frac{2.7 + 0.76}{1 + 0.76} \right) 9.8 = 19.3 \text{ kN/m}^3 \]
Stress Distribution-Tabular form

Step 2: Calculate the stresses using a table or use an MS Excel worksheet

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Thickness (m)</th>
<th>$\sigma_z$ (kPa)</th>
<th>$u$ (kPa)</th>
<th>$\sigma_z = \sigma - u$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$20.7 \times 2 = 41.4$</td>
<td>$-1 \times 9.8 = -9.8$</td>
<td>51.6</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>$41.4 + 22.4 \times 1 = 63.8$</td>
<td>0</td>
<td>63.8</td>
</tr>
<tr>
<td>5.4</td>
<td>2.4</td>
<td>$63.8 + 22.4 \times 2.4 = 117.6$</td>
<td>$2.4 \times 9.8 = 23.5$</td>
<td>94.1</td>
</tr>
<tr>
<td>20.6</td>
<td>15.2</td>
<td>$117.6 + 19.3 \times 15.2 = 411$</td>
<td>$23.5 + 15.2 \times 9.8 = 172.5$</td>
<td>238.5</td>
</tr>
</tbody>
</table>

Or $17.6 \times 9.8 = 172.5$
Step 3: Plot the stress versus depth

Stress Distribution - Graphical form

- Very fine wet sand with silt, $w = 5\%$, $S = 40\%$
- Fine sand saturated by capillary action
- Fine sand, $w = 12\%$
- Soft blue clay, $w = 28\%$

Graph showing stress (kPa) versus elevation (m) with different layers and their properties.
Stresses in Saturated Soil with Upward Seepage
Upward Seepage

• If water is moving or seeping, the effective stress at any point in a soil mass will differ from that in the static case.

• It will increase or decrease, depending on the direction of movement or seepage.

• The Figure shows a layer of granular soil in a tank where upward seepage is caused by adding water through the valve at the bottom of the tank.

• The rate of water supply is kept constant. The loss of head caused by upward seepage between the levels of A and B is \( h \).

• The total stress at any point in the soil mass is due solely to the weight of soil and water above it, we find that the effective stress calculations at points A and B are as follows:

\[
\sigma_{eff} = \frac{\gamma \cdot z}{\gamma_{w} \cdot \frac{h}{H_2}}
\]
At A,
- Total stress: $\sigma_A = H_1 \gamma_w$
- Pore water pressure: $u_A = H_1 \gamma_w$
- Effective stress: $\sigma'_A = \sigma_A - u_A = 0$

At B,
- Total stress: $\sigma_B = H_1 \gamma_w + H_2 \gamma_{sat}$
- Pore water pressure: $u_B = (H_1 + H_2 + h) \gamma_w$
- Effective stress: $\sigma'_B = \sigma_B - u_B$
  \[= H_2 (\gamma_{sat} - \gamma_w) - h \gamma_w\]
  \[= H_2 \gamma' - h \gamma_w\]

At C,
- Total stress: $\sigma_C = H_1 \gamma_w + z \gamma_{sat}$
- Pore water pressure: $u_C = \left( H_1 + z + \frac{h}{H_2} z \right) \gamma_w$
- Effective stress: $\sigma'_C = \sigma_C - u_C$
  \[= z (\gamma_{sat} - \gamma_w) - \frac{h}{H_2} z \gamma_w\]
  \[= z \gamma' - \frac{h}{H_2} z \gamma_w\]
Stress Profiles

Note that $h/H_2$ is the hydraulic gradient $i$ caused by the flow, and therefore

$$\sigma'_c = z\gamma' - iz\gamma_w$$

The variations of total stress, pore water pressure, and effective stress with depth are as given below:
Critical Hydraulic Gradient

If we compare the effective stress in upward seepage case with that without seepage at a point located at a depth $z$ measured from the surface of a soil layer.

The effective stress in upward seepage case is reduced by an amount $iz\gamma_w$ because of upward seepage of water.

If the rate of seepage and thereby the hydraulic gradient gradually are increased, a limiting condition will be reached, at which effective stress would become zero. At this point the hydraulic gradient is termed as Critical Hydraulic Gradient, $i_{cr}$.

$$\sigma'_c = z\gamma' - i_{cr}z\gamma_w = 0$$

Under this situation, soil stability is lost. This situation generally is referred to as boiling, or a quick condition

$$i_{cr} = \frac{\gamma'}{\gamma_w}$$

For most soils, the value of $i_{cr}$ varies from 0.9 to 1.1, with an average of 1.
Stresses in Saturated Soil with Downward Seepage
The hydraulic gradient caused by the downward seepage equals \( i = \frac{h}{H_2} \).

The total stress, pore water pressure, and effective stress at any point \( C \) are, respectively given as:

\[
\sigma_C = H_1 \gamma_w + z \gamma_{sat} \\
u_C = (H_1 + z - iz) \gamma_w \\
\sigma_C' = (H_1 \gamma_w + z \gamma_{sat}) - (H_1 + z - iz) \gamma_w \\
= z \gamma' + iz \gamma_w
\]
Stress Profiles

- Total stress, $\sigma$
  - $H_1 \gamma_w$
  - $H_1 \gamma_w + z \gamma_{sat}$
  - $H_1 \gamma_w + H_2 \gamma_{sat}$
- Pore water pressure, $u$
  - $H_1 \gamma_w$
  - $(H_1 + z - iz) \gamma_w$
- Effective stress, $\sigma'$
  - 0
  - $z \gamma' + iz \gamma_w$
  - $H_2 \gamma' + h \gamma_w$

Depth

(b) (c) (d)
Effect of Seepage direction on Effective Stress

Downward Seepage

No Seepage

Upward Seepage
Seepage Force

- As we have seen in the previous section that the effect of seepage is to increase or decrease the effective stress at a point in a layer of soil.

- Seepage force is conveniently expressed as force per unit volume of soil.

- From Figure 1, it can be seen that, with no seepage, the effective stress at a depth $z$ measured from the surface of the soil layer in the tank is equal to $zy'$. Thus, the effective force on an area $A$ is:

$$P_1' = zy'A$$

The direction of the force $P_1'$ is shown in Figure 2.
Again, in case of an upward seepage through the same soil layer, the effective force on an area $A$ at a depth $z$ can be given as (Figure 3):

$$P'_2 = (z\gamma' - iz\gamma_w)A$$

Hence, the decrease in the total force because of seepage is:

$$P'_1 - P'_2 = iz\gamma_wA$$

The volume of the soil contributing to the effective force equals $zA$, so the seepage force per unit volume of soil is:

$$\frac{P'_1 - P'_2}{\text{Volume of Soil}} = \frac{iz\gamma_wA}{zA} = i\gamma_w$$
Now, we can conclude that the seepage force per unit volume of soil is equal to $i\gamma_w$ and in isotropic soils the force acts in the same direction as the direction of flow.

- This statement is true for flow in any direction.
- The method of Flow nets can be used to compute the hydraulic gradient at any point and, thus, the seepage force per unit volume of soil.
Problem 4

In a tank, as shown in Figure, the upward flow of water through a layer of sand is taking place. The following properties of sand are given: void ratio, \( e = 0.52 \) and specific gravity of solids, \( G = 2.67 \).

a) Calculate the **total stress**, **pore water pressure**, and **effective stress** values at points A and B.

b) What is the upward seepage force per unit volume of soil?

**Solution: Part a**

The saturated unit weight of sand is calculated as follows:

\[
\gamma = \left( \frac{G_s + Se}{1 + e} \right) \gamma_w
\]

\[
= \frac{2.67(1 + 0.52)}{1 + 0.52} \times 9.8 = 20.59 \text{kN/m}^3
\]
Part b:

Hydraulic gradient, \( i = \frac{1.5}{2} = 0.75 \). Thus, the seepage force per unit volume can be calculated as:

\[
i \gamma_w = 0.75 \times 9.81 = 7.36 \text{kN/m}^3
\]
Capillary Saturation
Mechanics of Capillarity

• In silts and fine sands, the soil above the groundwater table are be saturated by capillary action.

• We can comprehend capillarity in soils by visualizing the continuous network of pore spaces as capillary tubes
Some Additional Concepts
Mechanics of Capillarity

The height to which water will rise in the tube can be computed from resolving forces. Summing forces vertically (downward positive), we get

\[ \sum F_z = \text{weight of water in the water column} - \text{tension forces due to capillary action} \]

\[ \frac{\pi d^2}{4} z_c \gamma_w - \pi d T \cos \theta = 0 \]

\[ z_c = \frac{4 T \cos \theta}{d \gamma_w} \]

Since \( T, \theta, \) and \( \gamma_w \) are constants, we can deduce

\[ z_c \propto \frac{1}{d} \]

Where, \( T \) is the surface tension (force per unit length), \( \theta \) = is the contact angle, \( z_c \) = is the height of capillary rise, and \( d \) = is the diameter of the tube representing the diameter of the void space.
Understanding capillarity for soils

In case of soils $d$ is taken to be equivalent to $0.1D_{10}$ where, $D_{10}$ is termed as effective size.

It implies that smaller the pore size, higher the capillary rise. For instance, capillary rise will be higher in case of fine sands as compared to medium sand, and same goes for medium sand relative to coarse sand.

Pore pressure in the capillary zone is negative as shown in Figure.

It is a function of pore size and water content.
Understanding capillarity for soils

- At the groundwater level, the porewater pressure is zero.
- It decreases (becomes negative) as we move up the capillary zone.
- The effective stress increases because the porewater pressure is negative.
- For example, for the capillary zone, $z_c$ porewater pressure at the top is $-z_c \gamma_w$

The increase in the effective stress is shown as:

$$
\sigma' = \sigma - u \\
= \sigma - (-z_c \gamma_w) \\
= \sigma + z_c \gamma_w
$$
Problem

A soil profile is shown in Figure. Calculate the total stress, pore water pressure, and effective stress at points A, B, and C.

At Point A:
- Total stress: \( \sigma_A = 0 \)
- Pore water pressure: \( u_A = 0 \)
- Effective stress: \( \sigma_A' = 0 \)

At Point B:
- \( \sigma_B = 6 \times \gamma_d \text{ (sand)} = 6 \times 16.5 = 99 \text{kN/m}^2 \)
- \( u_B = 0 \text{kN/m}^2 \)
- \( \sigma_A' = 99 - 0 = 99 \text{kN/m}^2 \)

At Point C:
- \( \sigma_C = 6 \times \gamma_d \text{ (sand)} + 13 \times \gamma_{sat} \text{ (sand)} \)
- \( = 6 \times 16.5 + 13 \times 19.25 \)
- \( = 99 + 250.25 = 349.25 \text{kN/m}^2 \)
- \( u_C = 13 \times \gamma_w = 13 \times 9.81 = 127.53 \text{kN/m}^2 \)
- \( \sigma_C' = 349.25 - 127.53 = 221.72 \text{kN/m}^2 \)